Calculus
Chapter 5 Group Competency

Name: KEY

Sketch the curve. Then, approximate the given integral using the **left-hand**, **right-hand, midpoint** and **trapezoid** methods with n subintervals.

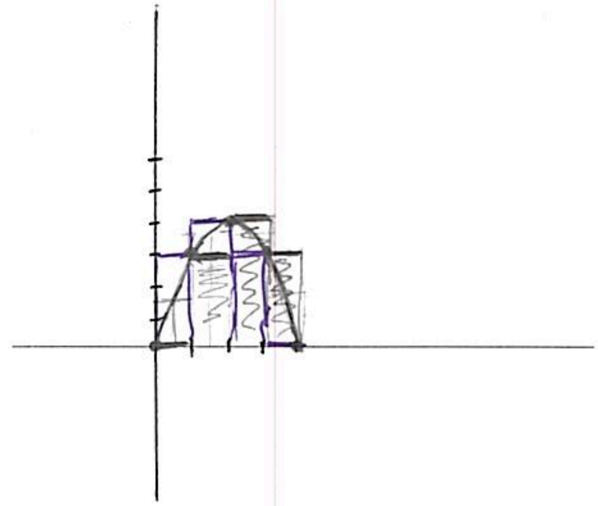
1. $\int_0^4 (4x - x^2)\,dx \quad n = 4$

L = $0\cdot 1 + 3\cdot 1 + 4\cdot 1 + 3\cdot 1 = 10$

R = $3 + 4 + 3 + 0 = 10$

M = $1.75 + 3.75 + 3.75 + 1.75 = 11$

T = 10



x	0	1	2	3	4
f(x)	0	3	4	3	0

Use **the definition as a limit** to find the area under the curve for the specified interval.

2. $f(x) = 4x - x^2 \qquad [0,4]$

$\Delta x = \frac{4}{n} \qquad x_k = \frac{4k}{n} \qquad f(x_k) = \frac{16k}{n} + \frac{16k^2}{n^2}$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{16k}{n} - \frac{16k^2}{n^2} \right) \frac{4}{n}$$

$$\lim_{n\to\infty} \sum \frac{64k}{n^2} - \sum \frac{64k^2}{n^3}$$

$$\lim_{n\to\infty} \frac{64}{n^2} \sum_{k=1}^{n} k - \frac{64}{n^3} \sum_{k=1}^{n} k^2$$

$$\lim_{n\to\infty} \frac{64}{n^2} \cdot \frac{n(n+1)}{2} - \frac{64}{n^3} \frac{n(n+1)(2n+1)}{6} = \lim_{n\to\infty} \frac{64\cancel{n^2}}{2\cancel{n^2}} - \frac{128\cancel{n^3}}{6\cancel{n^3}}$$

$$32 - \frac{64}{3} = \boxed{\frac{32}{3}}$$

Find each indefinite integral.

3. $\int \frac{4 - 2x}{x^3}\,dx$

$$\int (4x^{-3} - 2x^{-2})\,dx$$

$$\boxed{-2x^{-2} + 2x^{-1} + C}$$

OR

$$\boxed{\frac{-2}{x^2} + \frac{2}{x} + C}$$

4. $\int \cos x(\sec x + \tan x)\,dx$

$$\int \cos x \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x} \right) dx$$

$$\int (1 + \sin x)\,dx$$

$$\boxed{x - \cos x + C}$$

5. $\int \left(\frac{3}{x^2 + 1} - \frac{2}{x} \right) dx$

$$\boxed{3\tan^{-1} x - 2\ln|x| + C}$$

Find each indefinite integral using substitution.

6. $\int 2x\sqrt{4x^2+5}\,dx$

$u = 4x^2+5$

$du = 8x\,dx$

$\frac{1}{4}du = 2x\,dx$

$\frac{1}{4}\int u^{\frac{1}{2}}\,du$

$\frac{1}{4}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$\boxed{\frac{1}{6}(4x^2+5)^{\frac{3}{2}} + C}$

7. $\int \cos x \sin^5 x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\int u^5\,du$

$\frac{1}{6}u^6 + C$

$\boxed{\frac{1}{6}\sin^6 x + C}$

8. $\int x\left(\frac{x}{2}+3\right)^4 dx$

$u = \frac{x}{2}+3$

$du = \frac{1}{2}dx$

$2du = dx$

$x = 2(u-3)$

$\int 2(u-3)u^4\cdot 2du$

$\int (4u^5 - 12u^4)\,du$

$\frac{2}{3}u^6 - \frac{12}{5}u^5 + C$

$\boxed{\frac{2}{3}\left(\frac{x}{2}+3\right)^6 - \frac{12}{5}\left(\frac{x}{2}+3\right)^5 + C}$

If $\int_1^4 f(x)dx = 8$, $\int_1^2 f(x)dx = -5$, $\int_4^7 f(x)dx = -3$, and $\int_1^4 g(x)dx = -2$. Find each.

9. $\int_2^4 f(x)dx$

$\int_1^4 f(x)dx - \int_1^2 f(x)dx$

$8 - {}^{-}5 = \boxed{13}$

10. $\int_1^7 f(x)dx$

$\int_1^4 f(x)dx + \int_4^7 f(x)dx$

$8 + {}^{-}3 = \boxed{5}$

11. $\int_1^4 [f(x)+3g(x)]dx$

$\int_1^4 f(x)dx + 3\int_1^4 g(x)dx$

$8 + 3({}^{-}2) = \boxed{2}$

Solve each initial value problem.

12. $\frac{dy}{dx} = 3x^2 + x - 1$ $\quad (2,1)$

$y = x^3 + \frac{1}{2}x^2 - x + C$

$1 = 2^3 + \frac{1}{2}(2)^2 - 2 + C$

$1 = 8 + \cancel{2} - \cancel{2} + C$

${}^{-}7 = C$

$\boxed{y = x^3 + \frac{1}{2}x^2 - x - 7}$

13. $\frac{dy}{dx} = \cos x + \sin x$ $\quad y(0) = 1$

$y = \sin x - \cos x + C$

$1 = \sin 0 - \cos 0 + C$

$1 = 0 - 1 + C$

$2 = C$

$\boxed{y = \sin x - \cos x + 2}$

Evaluate the definite integrals using the Fundamental Theorem of Calculus.

14. $\int_{\frac{\sqrt{2}}{2}}^{1} \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1} x \Big]_{\frac{\sqrt{2}}{2}}^{1}$

$\sin^{-1} 1 - \sin^{-1}\frac{\sqrt{2}}{2}$

$\frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$

15. $\int_0^1 \left(7x^2\sqrt{x} - 2e^x\right)dx$

$\int 7x^{\frac{5}{2}} - 2e^x\,dx$

$2x^{\frac{7}{2}} - 2e^x \Big]_0^1$

$(2-2e) - (0-2) = \boxed{4-2e}$

Evaluate these definite integrals using substitution.

16. $\int_{-1}^{4} \frac{x}{\sqrt{x+5}}\,dx$

$u = x + 5$

$du = dx$

$x = u - 5$

$\int_4^9 (u-5)u^{-\frac{1}{2}}\,du$

$\int_4^9 \left(u^{\frac{1}{2}} - 5u^{-\frac{1}{2}}\right)du$

$\frac{2}{3}u^{\frac{3}{2}} - 10u^{\frac{1}{2}}\Big]_4^9$

$(18-30) - \left(\frac{16}{3} - 20\right)$

$8 - \frac{16}{3} = \boxed{\frac{8}{3}}$

17. $\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$\int_0^1 \sqrt{u}\,du$

$\frac{2}{3}u^{\frac{3}{2}}\Big]_0^1$

$\frac{2}{3} - 0 = \boxed{\frac{2}{3}}$

Use the information to find the velocity function. Then, find the displacement and the distance traveled by the particle during the given interval.

18. $a(t) = t - 2$, $v_0 = 0$, $1 \le t \le 5$

$v(t) = \frac{1}{2}t^2 - 2t + C$ $\quad C = 0$

$\boxed{v(t) = \frac{1}{2}t^2 - 2t}$

DISPLACEMENT

$\int_1^5 \left(\frac{1}{2}t^2 - 2t\right)dt$

$\frac{1}{6}t^3 - t^2\Big]_1^5$

$\frac{124}{6} - \frac{144}{6} = \frac{-20}{6}$

$\left(\frac{125}{6} - 25\right) - \left(\frac{1}{6} - 1\right) = \boxed{\frac{-10}{3}}$

DISTANCE

$\int_1^5 \left|\frac{1}{2}t^2 - 2t\right| dt = \boxed{5\frac{2}{3} = \frac{17}{3}}$

USE A CALCULATOR OR

$\boxed{-\int_1^4 \left(\frac{1}{2}t^2 - 2t\right)dt + \int_4^5 \left(\frac{1}{2}t^2 - 2t\right)dt}$

Find the average value of the function over the given interval.

19. $f(x) = \sqrt[3]{x}$ $\quad [-1, 8]$

$\frac{\int_{-1}^{8} x^{\frac{1}{3}}\,dx}{8 + 1} = \frac{\frac{3}{4}x^{\frac{4}{3}}\Big]_{-1}^{8}}{9} = \frac{12 - \frac{3}{4}}{9} = \frac{\frac{45}{4}}{9} = \boxed{\frac{5}{4}}$

$\frac{48}{4}$

1. $\int \sqrt[4]{x}\, dx =$ $\int x^{\frac{1}{4}} dx = \frac{4}{5}x^{\frac{5}{4}} + C$

A) $\dfrac{1}{4x^{3/4}} + C$

B) $-\dfrac{4}{5}x^{5/4} + C$

C) $-\dfrac{1}{4x^{3/4}} + C$

D) $\dfrac{4}{5}x^{5/4} + C$

E) $4x^{1/4} + C$

2. $2\int 8 \overset{\frac{1}{2}}{\sin} 4x\, dx =$

$u = 4x$

$du = 4dx$

$2du = 8dx$

$\int 2 \sin u\, du$

$-2\cos u + C$

A) 2 cos 4x + C

B) -2 cos x + C

C) -2 cos 4x + C

D) -2 sin²4x + C

E) 2 sin²4x + C

3. $\int \underline{5x^4} \left(x^5 - 5\right)^9 \underline{dx} =$

$u = x^5 - 5$

$du = 5x^4 dx$

$\int u^9 du$

$\frac{1}{10}(x^5-5)^{10} + C$

A) $\dfrac{\left(x^5 - 5\right)^{10}}{10} + C$

B) $\left(x^5 - 5\right)^{10} + C$

C) $\dfrac{\left(x^5 - 5\right)^{9}}{9} + C$

D) $\dfrac{\left(x^5 + 5\right)^{10}}{10} + C$

E) $\dfrac{5x\left(x^5 - 5\right)^{10}}{10} + C$

4. $\int_1^5 \dfrac{5}{x^2}\, dx =$ $\int_1^5 5x^{-2} dx$

$-5x^{-1}\Big]_1^5$

$\frac{-5}{5} - \frac{-5}{1}$

$-1 + 5 = 4$

A) 4

B) –1

C) –5

D) –6

E) 6

5. $\displaystyle\sum_{k=1}^{9} \sin(k\pi) =$

A) 9

B) 0

C) 18

D) 10

E) 8

6. Find the area under the curve $f(x) = e^{3x}$ over the interval [0, 3].

A) 2,702

B) 6

C) 9

D) 2,701

E) 8,103

$\int_0^3 e^{3x} dx$

$\frac{1}{3}\int_0^3 3e^{3x} dx$

$\frac{1}{3}e^{3x}\Big]_0^3 = \frac{1}{3}e^9 - \frac{1}{3} =$

7. Find the displacement of a particle if $v(t) = t^5 + 8$; [0, 6].

A) 46,704

B) 7,824

C) 3,173

D) 1,344

E) 7,784

$\int_0^6 (t^5 + 8)\, dt$

$\frac{1}{6}t^6 + 8t\Big]_0^6$

8. Find the average value of $f(x) = \dfrac{1}{x}$ over the interval [1, 4]. Approximate value to three decimal places.

A) 1.406

B) 18.199

C) 0.250

D) 0.462

E) 1.386

$\dfrac{\int_1^4 \frac{1}{x} dx}{4-1} = \dfrac{\ln|x|\Big]_1^4}{3} = \dfrac{\ln 4 - \ln 1}{3}$

$= \dfrac{\ln 4}{3}$