Calculus
Chapter 5 Review

Name: KEY

Approximate the given integral using the **left-hand** and **right-hand** methods with 2 subintervals. Then, use **the definition as a limit** to find the area under the curve for the specified interval.

1. $f(x) = 8 - 2x^2$ $[0,2]$ L = $8 + 6 = 14$ R = $6 + 0 = 6$

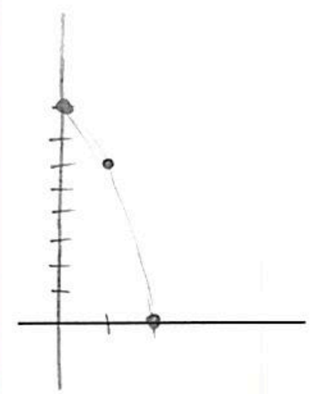
$\lim\limits_{n \to \infty} \sum\limits_{k=1}^{n} f(x_k)\Delta x$ $\Delta x = \frac{2}{n}$ $x_k = \frac{2k}{n}$

$= \lim\limits_{n \to \infty} \sum\limits_{k=1}^{n} \left(8 - \frac{8k^2}{n^2}\right)\frac{2}{n}$

$\lim\limits_{n \to \infty} \frac{2}{n}\left(\sum 8 - \frac{8}{n^2}\sum k^2\right)$

$\lim\limits_{n \to \infty} \frac{2}{n}\left(8n - \frac{8}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)\right)$

$16 - \frac{32}{6} = \frac{48}{3} - \frac{16}{3} = \boxed{\frac{32}{3}}$



Find each indefinite integral.

2. $\int \frac{2x^2 - x}{2\sqrt{x}}dx$ $\frac{1}{2}x^{-\frac{1}{2}}$

$\int \left(x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{1}{2}}\right)dx$

$\boxed{\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{3}x^{\frac{3}{2}} + C}$

3. $\int \left(\frac{7}{\sqrt{1-x^2}} + \frac{1}{x} - 3e^x\right)dx$

$\boxed{7\sin^{-1}x + \ln|x| - 3e^x + C}$

4. $\int \frac{\sec x + 5\cos x}{\cos x}dx$

$\int (\sec^2 x + 5)dx$

$\boxed{\tan x + 5x + C}$

Find each indefinite integral using substitution.

5. $\int \frac{x}{(x^2+2)^7}dx$ $\boxed{u = x^2 + 2}$ $du = 2x\,dx$ $\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int u^{-7}du$

$\frac{-1}{12}u^{-6} + C$

$\boxed{\frac{-1}{12(x^2+2)^6} + C}$

6. $\int \frac{\sin x}{1+\cos^2 x}dx$ $\boxed{u = \cos x}$ $du = -\sin x\,dx$

$-\int \frac{du}{1+u^2}$

$-\tan^{-1}u + C$

$\boxed{-\tan^{-1}(\cos x) + C}$

7. $\int x\sqrt{2x-1}\,dx$ $\frac{u+1}{2} = x$ $\boxed{u = 2x - 1}$ $du = 2dx$ $\frac{1}{2}du = dx$

$\frac{1}{4}\int (u+1)u^{\frac{1}{2}}du$

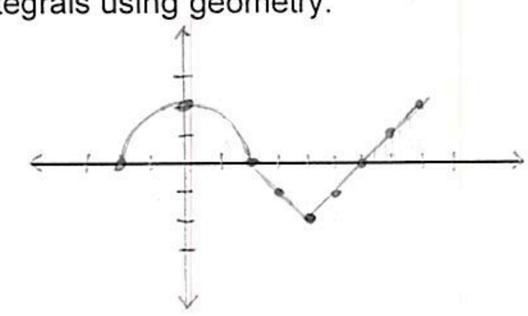
$\frac{1}{4}\int \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du$

$\frac{1}{4}\left(\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right) + C$

$\boxed{\frac{1}{10}(2x-1)^{\frac{5}{2}} + \frac{1}{6}(2x-1)^{\frac{3}{2}} + C}$

Sketch the piecewise function. Then, find the following definite integrals using geometry.

$$f(x) = \begin{cases} \sqrt{4-x^2}, & -2 \le x \le 2 \\ |x-4| - 2, & x > 2 \end{cases}$$



8. $\int_{-2}^{2} f(x)dx$ $\boxed{2\pi}$

9. $\int_{2}^{6} f(x)dx$ $\boxed{-4}$

10. $\int_{-2}^{8} f(x)dx$ $\boxed{2\pi - 2}$

Solve each initial value problem.

11. $\frac{dy}{dx} = \sec x \tan x + \sin x \qquad \left(\frac{\pi}{3}, 2\right)$

$y = \sec x - \cos x + C$

$2 = \sec\frac{\pi}{3} - \cos\frac{\pi}{3} + C$

$2 = 2 - \frac{1}{2} + C$

$\frac{1}{2} = C$

$\boxed{y = \sec x - \cos x + \frac{1}{2}}$

12. $\frac{dy}{dx} = 2e^{2x} \qquad y(0) = 5$

$y = e^{2x} + C$

$5 = e^{0} + C$

$C = 4$

$\boxed{y = e^{2x} + 4}$

Evaluate the definite integrals using the Fundamental Theorem of Calculus.

13. $\int_{-1}^{2} 4x(1-x^2)\,dx$

$\int_{-1}^{2} (4x - 4x^3)\,dx$

$2x^2 - x^4 \Big]_{-1}^{2}$

$(8 - 16) - (2 - 1)$

$-8 - 1 = \boxed{-9}$

14. $\int_{\frac{2}{\sqrt{3}}}^{2} \frac{dx}{x\sqrt{x^2-1}}\,dx$

$\sec^{-1} x \Big]_{\frac{2}{\sqrt{3}}}^{2}$

$\sec^{-1} 2 - \sec^{-1}\frac{2}{\sqrt{3}}$

$\frac{\pi}{3} - \frac{\pi}{6} = \boxed{\frac{\pi}{6}}$

15. $\int_{1}^{4} \left(\frac{2}{x^3} + \frac{1}{x\sqrt{x}}\right) dx$

$\int_{1}^{4} \left(2x^{-3} + x^{-\frac{3}{2}}\right) dx$

$-x^{-2} - 2x^{-\frac{1}{2}} \Big]_{1}^{4}$

$\frac{-1}{x^2} - \frac{2}{\sqrt{x}} \Big]_{1}^{4}$

$\left(\frac{-1}{16} - 1\right) - (-1 - 2)$

$\frac{-1}{16} + 2 = \boxed{\frac{31}{16}}$

Use the information to find the position function. Also, find the displacement and the distance traveled by the particle during the given interval.

16. $v(t) = 2t - 4, \quad s(1) = 2, \qquad 0 \le t \le 6$

$s(t) = t^2 - 4t + C$

$2 = 1 - 4 + C$

$5 = C$

$\boxed{s(t) = t^2 - 4t + 5}$

DISPLACEMENT

$\int_{0}^{6} (2t - 4)\,dt$

$t^2 - 4t \Big]_{0}^{6}$

$36 - 24 = \boxed{12}$

DISTANCE

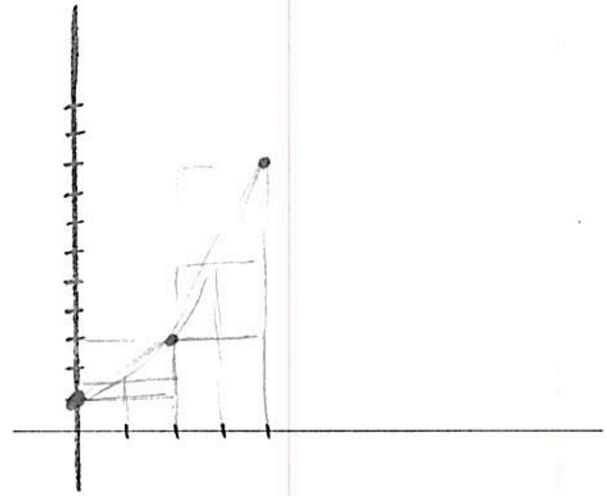
$\int_{0}^{6} |2t - 4|\,dt = \boxed{20}$

Calculus
Chapter 5 Extra Review

Name: KEY

Sketch the curve. Then, approximate the given integral using the **left-hand**, **right-hand, midpoint** and **trapezoid** methods with n subintervals.

1. $\int_0^4 \left(\frac{x^2}{2}+1\right) dx \qquad n=2$



L = $(1\cdot 2)+(3\cdot 2) = 8$

R = $(3\cdot 2)+(9\cdot 2) = 24$

M = $(1.5\cdot 2)+(5.5\cdot 2) = 14$

T = $\frac{8+24}{2} = 16$

Use **the definition as a limit** to find the area under the curve for the specified interval.

2. $f(x) = \frac{x^2}{2}+1 \qquad [0,4]$

$\Delta x = \frac{4}{n}$

$x_k = \frac{4k}{n}$

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{16k^2}{2n^2}+1\right)\frac{4}{n}$$

$$\lim_{n\to\infty} \frac{4}{n}\left(\frac{8}{n^2}\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 1\right)$$

$$\lim_{n\to\infty} \frac{4}{n}\left(\frac{8}{n^2}\cdot\frac{n(n+1)(2n+1)}{6} + 1n\right)$$

$$\lim_{n\to\infty} \left(\frac{64n^3}{6n^3} + \frac{4n}{n}\right)$$

$$\frac{32}{3} + \frac{12}{3} = \boxed{\frac{44}{3}}$$

Find this indefinite integral using substitution.

3. $\int x\sqrt{2x-1}\,dx$

$u = 2x - 1 \longrightarrow x = \frac{u+1}{2}$

$du = 2dx$

$\int \frac{u+1}{2}\sqrt{u}\,\frac{du}{2}$

$\frac{1}{4}\int (u+1)u^{\frac{1}{2}}\,du$

$\frac{1}{4}\int \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du$

$\frac{1}{4}\left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right] + C = \boxed{\frac{1}{10}(2x-1)^{\frac{5}{2}} + \frac{1}{6}(2x-1)^{\frac{3}{2}} + C}$

Evaluate the definite integrals using the Fundamental Theorem of Calculus.

4. $\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$

$\sec^{-1} x\Big]_{\sqrt{2}}^{2}$

$\sec^{-1} 2 - \sec^{-1}\sqrt{2}$

$\frac{\pi}{3} - \frac{\pi}{4} = \boxed{\frac{\pi}{12}}$

Use the information to find the velocity function. Then, find the distance traveled by the particle during the given interval.

5. $a(t) = \sin t,\quad v_0 = 1,\qquad 0 \le t \le \pi$

$v(t) = -\cos t + C$

$1 = -1 + C$

$2 = C$

$\boxed{v(t) = -\cos t + 2}$

DISTANCE $= \int_0^{\pi} \left| -\cos t + 2 \right| dt = \boxed{2\pi}$

$-\sin t + 2t\Big]_0^{\pi} = 2\pi$

$(2\pi - 0)$